Baryon Mass Splitting and Weak and Strong Couplings*†

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The mass difference between various baryons is attributed to the presence of bosons weakly coupled to a primary baryon. As an example, the case of $\Sigma\Lambda$ is treated assuming the $\Sigma-\Lambda$ parity to be even. An eigenvalue condition derived by Albright, Blankenbecler, and Goldberger, using the N/D method, is rederived simply. This derivation not only clarifies the assumptions, but makes the algebraic handling of the problem very simple.

1. INTRODUCTION

STRONG interactions may have their origin in weak interactions; several attempts have been made recently to examine this hypothesis.[1,2] Thirring[1] has considered the pion as a bound state of a nucleon-antinucleon pair (as in the Fermi-Yang model[3]). By a beautiful exercise of analytical skill, he has solved the two-fermion equation[4] in the limit of large binding energy and produced a pion whose binding force is entirely due to the exchange of a massive vector boson interacting weakly with the nucleons. In this calculation, crossing symmetry is violated because the nature of the approximation gives rise to a pion interacting much more strongly with the nucleon than do the vector mesons themselves. To remedy this defect, Albright, Blankenbecler, and Goldberger[2] have done a dispersion-theoretic calculation in which the vector boson and the pion are treated on the same footing.

The present paper might be considered as a logical continuation of the work of Albright *et al.*[2] There have been many symmetry schemes of elementary particles which start out with baryons of equal mass (a degenerate baryon) and break down the symmetries to account for the observed mass differences. The breakdown of the symmetry is attributed to pions and kaons. In this work, we attribute the mass differences between various baryons to the presence of vector bosons. We think of the baryons as bound states of a nucleon and a pion or a K meson in the presence of weakly interacting vector bosons. As a simple example, we consider the coupled system of a Λ particle and a pion to produce a Σ particle.

In the next section, we derive two eigenvalue equations connecting masses of the particles involved in the two basic vertices and the vertex values themselves, that is, the coupling constants. This derivation does not make use of the N/D method and the special asymptotic behavior of the vertex function assumed by Albright *et al.* Thus, it makes the construction of the matrices $\mathbf{M}$, $\mathbf{H}$, and $\boldsymbol{\rho}$ of reference 2 unnecessary. In Secs. 3 and 4, we treat the two cases: the weakly coupled boson as a pseudoscalar particle, and as a vector particle. The last section contains numerical results and discussion.

2. EIGENVALUE CONDITIONS

We define the two vertex functions G and H:

$$G=\langle 0|f_\Sigma|\Lambda V\rangle(2E_1k_0/M)^{1/2}, \tag{1a}$$

and

$$H=\langle 0|f_\Sigma|\Lambda\pi\rangle(2E_2q_0/M)^{1/2}, \tag{1b}$$

where the 4-momentum $(0,W)$ of the Σ particle is off the mass shell. f_Σ is the current operator for the Σ particle. (p,E), (k,k_0), and (q,q_0) are the 4-momenta and energy of the Λ particle, the V meson, and the pion. M_Σ, M, m, and μ are the masses of Σ, Λ, V, and π. We assume that the V meson has isotopic spin unity and that parity is conserved in all the interactions we consider. The subscripts 1 and 2 refer to the vertices involving V and π mesons, respectively. We assume in the following sections that the Σ-Λ parity is even and that the $\Sigma\pi\Sigma$ coupling is small, as indicated by analyses of the recent experimental data.[5]

We approximate the unitarity conditions for the vertex functions G and H by considering only one-meson and one-baryon intermediate states (Fig. 1). In this approximation, the unitarity conditions can be written as

$$\mathrm{Im}G=G^*\rho_{11}M_{11}+H^*\rho_{22}M_{21}, \tag{2a}$$

$$\mathrm{Im}H=G^*\rho_{11}M_{12}+H^*\rho_{22}M_{22}, \tag{2b}$$

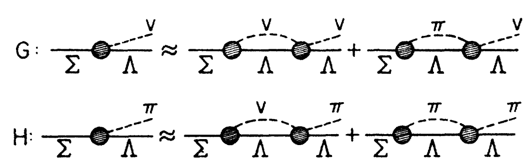

FIG. 1. Diagrammatic representation of the approximation to the unitarity condition.

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where ϱ is a diagonal matrix containing phase-space factors and $\mathbf{M}$, the scattering matrix.

The dispersion relations for the vertex functions are

$$G(W^2)=\frac{1}{\pi}\int^{\infty} dW'^2\frac{\mathrm{Im}G(W'^2)}{W'^2-W^2-i\epsilon}, \tag{3a}$$

$$H(W^2)=\frac{1}{\pi}\int^{\infty} dW'^2\frac{\mathrm{Im}H(W'^2)}{W'^2-W^2-i\epsilon}. \tag{3b}$$

The corresponding lower limits are to be inserted.

Now we assume that, because of the weakness of the V-meson coupling (and its presumably heavy mass[6]), the pion intermediate state predominates and the V-meson intermediate state contribution can be neglected. Then, from (2) and (3) we get

$$G(W^2)=\frac{1}{\pi}\int_{(M+\mu)^2}^{\infty} dW'^2\frac{H^*\rho_{22}M_{21}}{W'^2-W^2-i\epsilon}, \tag{4a}$$

$$H(W^2)=\frac{1}{\pi}\int_{(M+\mu)^2}^{\infty} dW'^2\frac{H^*\rho_{22}M_{22}}{W'^2-W^2-i\epsilon}. \tag{4b}$$

On the mass-shell of the Σ particle we have

$$g_v=\frac{1}{\pi}\int_{(M+\mu)^2}^{\infty} dW'^2\frac{H^*\rho_{22}M_{21}}{W'^2-M_\Sigma^2-i\epsilon},$$

$$g_\pi=\frac{1}{\pi}\int_{(M+\mu)^2}^{\infty} dW'^2\frac{H^*\rho_{22}M_{22}}{W'^2-M_\Sigma^2-i\epsilon}.$$

Defining:

$$I_{i2}=\frac{8\pi}{g_i g_\pi^2}\int_{(M+\mu)^2}^{\infty} dW'^2\frac{H^*\rho_{22}M_{2i}}{W'^2-M_\Sigma^2-i\epsilon}, \tag{5}$$

with $g_1=g_v$ and $g_2=g_\pi$, we have

$$g_\pi^2/4\pi=2\pi/I_{12}, \tag{6a}$$

and

$$g_\pi^2/4\pi=2\pi/I_{22}. \tag{6b}$$

If we assume the energy independence of $H^*(W'^2)$ inside the integral in (5) and setting $H^*(W'^2)=g_\pi$ and replacing $M(W'^2)$ by the first Born approximation $B(W'^2)$, Eq. (5) becomes

$$\begin{aligned} I_{i2}&=\frac{8\pi}{g_i g_\pi}\int_{(M+\mu)^2}^{\infty} dW'^2\frac{\rho_{22}(W'^2)B_{2i}(W'^2)}{W'^2-W^2-i\epsilon} \\ &=\frac{8\pi}{g_i g_\pi}\int_{(M+\mu)}^{\infty} dW'\left[\frac{\rho_{22}(W)B_{2i}(W)}{W'-W-i\epsilon}\right. \\ &\qquad\left.-\frac{\rho_{22}(-W)B_{2i}(-W)}{W'+W+i\epsilon}\right]. \end{aligned} \tag{7}$$

With this expression for I_{i2}, (6a) is exactly the same as the Eq. (2.25) obtained by Albright *et al.*[2] In addition, we have here (6b), which implies that, if the weak V meson is pseudoscalar, in this approximation, it should have the same mass as the pion. The comparison of the two methods of derivation indicates that the assumption about the asymptotic behavior of the vertex function in reference 2 is equivalent to neglecting the V-meson intermediate states in the unitarity condition.

Because of the nature of the present derivation of (6), we do not need to construct the $\mathbf{H}$, $\mathbf{M}$, and ϱ matrices of reference 2 and the vector V-meson case does not become any more difficult than the case of a pseudoscalar or scalar V meson.

Some remarks about Eqs. (6) are in order. If we take M and μ as known and g_π, m, and M_Σ as unknown quantities, we can, in principle, solve the two equations for one of the latter, say M_Σ, and get the other two, m and g_π, as a function of each other. The validity of our approximations, however, is much greater for Eq. (6a) than for (6b), so that a complete solution of the system of equations is not attempted here. This point is more fully discussed in the last section.

3. PSEUDOSCALAR V MESON

Assuming the Λ-Σ parity to be even, the invariant expression for G in the center-of-mass system of the V-Λ state is

$$\begin{aligned} G&=-i\{G_1+G_2[i\gamma\cdot(p+k)+M_\Sigma]\}\gamma_5 u(p), \\ &=-\tfrac{1}{2}i[G_+(1+\gamma_4)+G_-(1-\gamma_4)]\gamma_5 u(p), \end{aligned}$$

with

$$G_\pm=G_1+(M_\Sigma\mp W)G_2,$$

whence, in two-component notation

$$G=-i[2M(E_1+M)]^{-1/2}\begin{bmatrix} G_+\boldsymbol{\sigma}\cdot\mathbf{k} \\ -G_-(E_1+M)\end{bmatrix}\chi_1. \tag{8}$$

The notation is the same as in reference 2. For the $\Lambda+\pi\to\Lambda+V$ reaction, we have

$$\begin{aligned} T_{12}&=\bar{u}(p')[A_{12}-\tfrac{1}{2}i\gamma\cdot(q+k')B_{12}]u(p), \\ &=\frac{4\pi W}{M}\chi_1'^*[t_1+t_2\boldsymbol{\sigma}\cdot\hat{k}\boldsymbol{\sigma}\cdot\hat{q}]\chi_2, \end{aligned} \tag{9}$$

where

$$t_1=\frac{[(W+M)^2-\mu^2]^{1/2}[(W+M)^2-m^2]^{1/2}}{16\pi W^2}\times[A_{12}+(W-M)B_{12}],$$

$$t_2=\frac{[(W-M)^2-\mu^2]^{1/2}[(W-M)^2-m^2]^{1/2}}{16\pi W^2}\times[-A_{12}+(W+M)B_{12}].$$

Then the $(1-)$ and $(0+)$ partial waves are given by

$$\begin{aligned} t_{1-}&=t_1^1+t_2^0, \\ t_{0+}&=t_1^0+t_2^1, \end{aligned} \tag{10}$$

[6] T. D. Lee and C. N. Yang, Phys. Rev. **119**, 1410 (1960).

where t_i^l are coefficients in the partial-wave expansion of t_i,

$$t_i=\sum_l (2l+1)t_i^l P_l(\hat{q}'\cdot\hat{q}).$$

We see that G_+ on the mass shell is just g_v. Hence, we are interested in the dispersion relation for G_+. The unitarity condition (neglecting V-meson intermediate states) is

$$\mathrm{Im}G_+\approx H_+^* q[(E_2-M)/(E_1-M)]^{1/2}t_{1-},$$

which gives

$$\rho_{22}M_{21}\approx q[(E_2-M)/(E_1-M)]^{1/2}t_{-1}. \quad (11)$$

We approximate t_{1-} by the $(1-)$ partial-wave Born amplitude for $N+V\to N+\pi$. The interaction Lagrangian is

$$L_I=ig_v\bar{\psi}_\Sigma\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\phi}_v\psi_v-ig_\pi\bar{\psi}_\Sigma\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\phi}_\pi\psi_\Lambda. \quad (12)$$

The Born approximation for T_{21} is given by

$$T_{21}=-g_v g_\pi \bar{u}(p')\left[\frac{i\gamma\cdot(p+k)+M_\Sigma}{M_\Sigma^2-W^2}+\frac{i\gamma\cdot(p-q')+M_\Sigma}{(p-q')^2+M_\Sigma^2}\right]u(p). \quad (13)$$

Hence,

$$t_1=\frac{(E_1+M)^{1/2}(E_2+M)^{1/2}}{8\pi W}\times[\alpha(W-M_\Sigma)-\beta(W+M_\Sigma-2M)], \quad (14a)$$

$$t_2=\frac{(E_1-M)^{1/2}(E_2-M)^{1/2}}{8\pi W}\times[\alpha(W+M_\Sigma)-\beta(W-M_\Sigma+2M)], \quad (14b)$$

with

$$\alpha=g_v g_\pi/(M_\Sigma^2-W^2),$$

and

$$\beta=g_v g_\pi/[M_\Sigma^2+(p-q')]^2.$$

Projecting out t_1^1 and t_2^0 from Eq. (14) and combining (11) with (7), we get

$$I_{21}=M_\Sigma\int_{(M+\mu)}^\infty \frac{dW}{W}q(W)J_{ps}(W), \quad (15a)$$

where

$$J_{ps}=-\frac{1}{k^2}\left[\left(1-\frac{\Delta}{M_\Sigma}\right)+\frac{2\Delta}{W^2-M_\Sigma^2}\left(M_\Sigma-\Delta+\frac{WE_1}{M_\Sigma}\right)\right](1-ax)+\frac{2}{(W^2-M_\Sigma^2)}(\mu^2-\Delta^2)\left[1+(W^2-M_\Sigma^2)\left(1-\frac{\Delta}{M_\Sigma}\right)x\right]+\frac{2\Delta}{M_\Sigma}\frac{1}{W^2-M_\Sigma^2}, \quad (15b)$$

with

$$x=\frac{1}{2b}\ln\left(\frac{a+b}{a-b}\right),$$
$$\Delta=M_\Sigma-M, \quad (15c)$$
$$a=2E_2k_0-m^2+2\Delta M_\Sigma-\Delta^2,$$
$$b=2kq.$$

4. VECTOR V MESON

The vertex function G can be written as

$$G=-i\{G_1+G_2[i\gamma\cdot(p+k)+M_\Sigma]\}\gamma\cdot\xi u(p)+\{G_3+G_4[i\gamma\cdot(p+k)+M_\Sigma]\}\xi\cdot p u(p), \quad (16)$$

where ξ_μ is the polarization four-vector of the vector V meson and satisfies the equation

$$\xi\cdot k=0.$$

In the two-component notation Eq. (16) takes the form:

$$G=[2M(E_1+M)]^{-1/2}\begin{bmatrix}(F_++G_+)\boldsymbol{\xi}\cdot\mathbf{k}+G_+ i\boldsymbol{\sigma}\cdot\boldsymbol{\xi}\times\mathbf{k}\\ \{[F_-+(E_1+M_\Sigma)G_-]\boldsymbol{\xi}\cdot\mathbf{k}+(E_1+M_\Sigma)G_- i\boldsymbol{\sigma}\cdot\boldsymbol{\xi}\times\mathbf{k}\}\boldsymbol{\sigma}\cdot\mathbf{k}\end{bmatrix}\chi_1, \quad (17)$$

with

$$G_\pm=G_1+(M_\Sigma\mp W)G_2,$$
$$G_\pm'=G_3+(M_\Sigma\mp W)G_4,$$
$$F_+=(G_+-WG_+')(E_1+M_\Sigma)/(W-E_1),$$

and

$$F_-=(G_-+WG_+')(E_1^2-M_\Sigma^2)/(W-E_1).$$

In the two-component notation, T_{12} can be written as

$$T_{12}=i\frac{4\pi W}{W}\chi_1'^*[t_1\boldsymbol{\sigma}\cdot\boldsymbol{\xi}^*+t_2(i\boldsymbol{\sigma}\cdot\hat{k}\times\boldsymbol{\xi}^*)\boldsymbol{\sigma}\cdot\hat{q}+t_3\boldsymbol{\sigma}\cdot\hat{k}\boldsymbol{\xi}^*\cdot\hat{q}+t_4\boldsymbol{\sigma}\cdot\hat{q}\boldsymbol{\xi}^*\cdot\hat{q}+t_5\boldsymbol{\sigma}\cdot\hat{k}\boldsymbol{\xi}^*\cdot\hat{k}+t_6\boldsymbol{\sigma}\cdot\hat{q}\boldsymbol{\xi}^*\cdot\hat{k}]\chi_2. \quad (18)$$

The unitarity condition in the approximation in which we are working gives

$$\mathrm{Im}G_+=H_+^* q\left(\frac{E_2-M}{E_1-M}\right)^{1/2}(t_1^1-t_2^0-\tfrac{1}{3}t_3^0+\tfrac{1}{3}t_3^2), \quad (19)$$

t_i^l being the coefficients in the partial-wave expansions of t_i's of Eq. (18). The interaction Lagrangian in this case is

$$L_I=-g_v\bar{\psi}_\Sigma i\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\phi}_{v_\mu}\psi_\Lambda-g_\pi i\bar{\psi}_\Sigma\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\phi}_\pi\psi_\Lambda, \quad (20)$$

and the Born amplitude is given by

$$T_{21}=g_v g_\pi \bar{u}(p')\left[\frac{i\gamma\cdot(p+k)+M_\Sigma}{M_\Sigma^2-W^2}\gamma\cdot\xi - \gamma\cdot\xi\frac{i\gamma\cdot q+(M_\Sigma-M)}{(p-q')^2+M_\Sigma^2}\right]\gamma_5 u(p). \quad (21)$$

Putting the above expression in the form of Eq. (18), we get

$$t_1=\frac{1}{8\pi W}[(E_1+M)(E_2+M)]^{1/2}\times[\alpha(W-M_\Sigma)+\beta(W+M_\Sigma-2M)],$$
$$t_2=-\frac{1}{8\pi W}[(E_1-M)(E_2-M)]^{1/2}\times[\alpha(W-M_\Sigma)+\beta(W-M_\Sigma+2M)],$$
$$t_3=\frac{1}{8\pi W}[(E_1-M)(E_2-M)]^{1/2}2(E_2+M)\beta,$$
$$t_4=-\frac{1}{8\pi W}[(E_1+M)(E_2+M)]^{1/2}2(E_2-M)\beta, \quad (22)$$
$$t_5=\frac{1}{8\pi W}[(E_1+M)(E_2+M)]^{1/2}\frac{E_1-M}{W-E_1}\times[\alpha(W-M_\Sigma)+\beta(2E_2-W-2M+M_\Sigma)],$$
$$t_6=\frac{1}{8\pi W}[(E_1-M)(E_2-M)]^{1/2}\frac{1}{W-E_1}\times\{-(W+M)(W+M_\Sigma)\alpha-\beta[2E_2(E_1+M)+m^2+(W+M)(M-M_\Sigma)]\}.$$

Projecting the relevant angular momentum parts from Eqs. (22) and combining (19) with (7), we get

$$I_{12}=M_\Sigma\int_{(M+\mu)}^{\infty}dW\frac{q(W)}{W}J_v(W),$$

where

$$J_v=-J_{ps}+\frac{4(\mu^2-\Delta^2)}{(W^2-M_\Sigma^2)^2}-\frac{a}{2k^2(W^2-M_\Sigma^2)}-\frac{2q^2}{W^2-M_\Sigma^2}\left(1-\frac{a^2}{b^2}\right)x, \quad (24)$$

a, b, and x being given by (15c).

5. NUMERICAL CALCULATIONS AND DISCUSSION

Now, using Eq. (6b), we calculate $g_\pi^2/4\pi$ for various mass values of m, assuming M_Σ, M, and μ as known. Because we have used the Born approximation and neglected higher mass intermediate states, we calculate the values of the integral in I_{12} with various cutoff values to compare with the one obtained with infinite limit (Table I).

TABLE I. Calculation of $g^2/4\pi$ with various cutoff energies and various values of mass m.

Type of V meson	Mass m in BeV	$g_\pi^2/4\pi$ with cutoff at: $2M$	$3M$	∞
Pseudoscalar	0.14	50.1	26.2	5.6
	0.34	−46.8	−351.0	7.3
	0.74	−19.8	−32.0	9.1
	1.14	−15.2	−22.1	10.3
Vector	0.14	−53.5	−34.1	−6.2
	0.34	46.7	349.0	−8.0
	0.74	19.6	31.8	−9.8
	1.14	15.0	21.9	−11.1

In the pseudoscalar case, we see that the coupling constant is positive and increases with increasing V-particle mass for infinite cutoff. This is the opposite of the situation in reference 2. From an examination of the expression (15) and the corresponding expression in reference 2, we see that this difference in sign can be traced back to the differences in the isotopic spins of the participating particles.[7]

When the V meson is vector, we see that the values of $g^2/4\pi$ with infinite limit decrease as m is increased. Comparison with reference 2 again shows that the values we have are opposite in sign, the reason being the same as in the case of the pseudoscalar V meson. We can get positive values for $g^2/4\pi$ with a high cutoff value for the integral and with a higher value of m (see Table I).

As mentioned in Sec. 2, when the V meson is pseudoscalar, it should have the same mass as the pion. This follows from the identical functional dependence in Eqs. (6a) and (6b) when m is equal to μ. Because of the nature of the approximations we have used, the functional dependence in (6b) should not be taken seriously. The Born approximation could well be valid in the case of T_{21}, since one of the particles is weakly coupled to the Λ particle. Thus, although we have two eigenvalue conditions and in principle we can get $g^2/4\pi$ as a function of m, the nature of the approximation we have made, especially in deriving I_{22} in (6b), does not permit us to get a reliable functional dependence. We could do better by giving a dispersion-theoretic treatment to the Λ-π scattering amplitude, using the Mandelstam representation.

We see from the Table I that for the given value of M_Σ there exists a V meson mass with an appropriate cutoff value for the integral which gives a reasonable value for the coupling constant g_π. Thus the results, though not conclusive, make very plausible the idea that the mass difference between baryons may be due to the presence of various vector mesons.

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[7] One of the authors (K.T.M.) wishes to thank Dr. R. Blankenbecler for a discussion on this point.